

REG. NO

## JAYA GROUP OF INSTITUTIONS

2<sup>ND</sup> SEM - B.E

## INTERNAL ASSESSMENT - 3 (MODEL EXAMINATION- II)

Subject Name: Probability And Random Process

Subject Code: MA645.

Duration : 180 Minutes

Date : 11/04/2015

Branch : Common to All

Max Marks: 100

PART - A (10x2=20) answer all the Questions:

- Establish the memory less property of the exponential distribution.
- Find C If  $P(X = n) = C \left(\frac{2}{3}\right)^n$   $n = 1, 2, 3, \dots$
- Let  $X$  and  $Y$  be Continuous RVs with Joint pdf  $f_{XY}(x, y) = \begin{cases} \frac{x(x-y)}{8}, & 0 < x < 2, -x < y < x \\ 0, & \text{otherwise} \end{cases}$  find  $f_{X/Y}(x/y)$
- Give a real life example each for positive correlation and negative correlation
- Define: Markov Process
- Define Random telegraph signal Process.
- Find the mean and variance the stationary process  $\{X(t)\}$  whose ACF IS  $R(\tau) = 16 + \frac{9}{1+16\tau^2}$
- Prove that for a WSS Process  $\{X(t),\}$   $R_{XX}(t, t + \tau)$  Is Even function of  $\tau$ .
- Find system Transfer Function if the linear time invariant system has an impulse function  $h(t) = \begin{cases} \frac{1}{2c}, & |t| < c \\ 0, & |t| \geq c \end{cases}$
- Define: Time invariant System

PART - B (5 X 16 = 80) Answer all the Questions as per the Choice:

- a) The pdf of a R.V is given by  $f(x) = \begin{cases} x, & 0 < x < 1 \\ k(2-x), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ 
  - Find the value of k
  - Find  $P(0.2 < X < 1.2)$
  - Find the Distribution Function of X
  - Find  $P(0.5 < X < 1.5/X \geq 1)$  (8)
- (i) Derive the Mean, Variance, and MGF of the Binomial Distribution. (8)

(OR)

- Given that  $X$  is distributed normally if  $P(X < 45) = 0.31$  and  $P(X > 64) = 0.08$  find the mean and standard deviation of the distribution. (8)

- A Random Variable  $X$  has the following probability distribution function

|      |   |   |    |    |    |       |        |          |
|------|---|---|----|----|----|-------|--------|----------|
| x    | 0 | 1 | 2  | 3  | 4  | 5     | 6      | 7        |
| P(x) | 0 | K | 2K | 2k | 3K | $k^2$ | $2k^2$ | $7k^2+k$ |

- Find the value of k
- Evaluate  $P(X < 6)$ ,  $P(X \geq 6)$
- If  $P(X \leq c) > \frac{1}{2}$  find the minimum value of c. (8)

12. a) The joint pdf of the RV  $(X, Y)$  is given by  $f(x, y) = kxye^{-(x^2+y^2)}$ ,  $x > 0, y > 0$  find the value of  $k$  (8)  
 (i) And  $\text{COV}(X, Y)$ . Are  $X$  and  $Y$  independent?

(ii) If  $X$  and  $Y$  are independent RVs with density function,  $f_x(x) = \begin{cases} 1, & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$ ,  
 $f_y(y) = \begin{cases} \frac{y}{6}, & 2 \leq y \leq 4 \\ 0, & \text{otherwise} \end{cases}$  Find the density function of  $Z = XY$  (8) (OR)

- b(i) The Joint p.d.f of  $X$  and  $Y$  is given by  $f(x, y) = \begin{cases} 2 - x - y, & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{if otherwise} \end{cases}$

Show that  $\text{Cov}(X, Y) = -\frac{1}{144}$  (8)

- (ii) Let  $X$  and  $Y$  be two R.V with the Joint p.d.f  $f(x, y) = xy^2 + \frac{x^2}{8}$ ,  $0 \leq x \leq 2, 0 \leq y \leq 1$

Compute i.  $P(X > 1 | Y = \frac{1}{2})$  ii.  $P(X + Y \leq 1)$  iii.  $P(X < Y)$  (8)

13. a) Examine whether the random process  $\{X(t)\} = A \cos(\omega t + \theta)$  is a WSS if  $A$  and  $\omega$  are constants and  $\theta$  is uniformly distributed in  $(0, 2\pi)$  (8)

- (i) Derive the probability law of Poisson process (8)  
 (OR)

- b(i) If the WSS process  $\{X(t)\}$  is given by  $X(t) = 10 \cos(100t + \theta)$  where  $\theta$  is uniformly distributed over  $(-\pi, \pi)$  prove that  $\{X(t)\}$  is correlation ergodic. (8)

- (ii) Find the Nature of the states of the markov chain with the tpm  $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$  (8)

14. a) State and prove any four properties of autocorrelation function. (8)

- (i) Find the autocorrelation function and average power of the process  $S_{XX}(\omega) = \frac{1}{(1+\omega^2)^2}$  (8)  
 (OR)

- b(i) The cross-power spectrum of a read R.P  $\{X(t)\}$  and  $\{Y(t)\}$  is

$s_{xy}(\omega) = \begin{cases} a + bj\omega, & \text{for } |\omega| < 1 \\ 0, & \text{otherwise} \end{cases}$  Find the cross correlation function. (8)

- (ii) For the Process  $X(t) = a \cos(bt + y)$  where  $y$  is the normally distributed over  $(0, 2\pi)$  find the autocorrelation function and power spectral density. (8)

15. a) Show that if the input  $\{X(t)\}$  is a WSS process for a linear system then the output  $\{Y(t)\}$  is a WSS (8)

- (i) A system has an impulse response  $h(t) = e^{-\beta t} U(t)$  find the PSD for the  $\{Y(t)\}$  corresponding to the Input  $\{X(t)\}$  (8)

(OR)

- b(i) Let  $X(t)$  be a WSS process which is the input to a linear time invariant system with unit impulse  $h(t)$  and the Output  $Y(t)$  and then prove that  $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$  (8)

- (ii) Assume a R.P  $X(t)$  is input to a system transfer function  $H(\omega) = 1, -\omega_0 < \omega < \omega_0$  if the autocorrelation of the input Process  $\frac{N_0}{2} \delta(t)$  find the auto correlation of the output process. (8)

ALL THE BEST