JAYA GROUP OF INSTITUTIONS

THIRUNINRAVUR

4^{TH}-SEMSETER-B.E

MODEL EXAMINATION-I(27.01.2015 TO 02.01.2015)

SUB CODE: MA6451 | DATE :27.01.2015

SUBJECT TITLE: PROBABILITY AND RANDOM PROCESSES | BRANCH : ECE

DURATION : 3 HRS | MAX.MARK:100

PART-A (10*2=20)

Answer all the questions.

1.Define discrete random variable.

2.Define geometric distribution.

3. State and prove additive property of binomial distribution.

4.A continuous RV X has probability density function f(x)=$3x^2$, $0 \le x \le 1$ .Find k such that P(X>k)=0.05.

5.If a random variable X has the MGF, $M_x(t) = \frac{2}{2-t}$,find the variance of 'X'.

6. Define markov process.

7. Define WSS process

8.Show that the random process X(t)=Asin($X(t) = A\sin(\varpi t + \phi)$,where A and ϖ are constants, ϕ is a random variable uniformly distributed in (0.2π) is first order stationary.

9. State the postulates of a Poisson process.

10. Consider the random process $X(t) = \cos(t+\phi)$, where ϕ is a random variable with density function $f(\phi) = \frac{1}{\pi}$, $\frac{-\pi}{2} < \phi < \frac{\pi}{2}$. Check whether the process is stationary or not.

Part-B(5*16=80)

11(a)(i) A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find (a) The value of k

(b) p(1.5<X<4.5/x>2)

(c) The smallest value of n for which $p(x \le n) > \frac{1}{2}$

(ii) A random variable X has pdf $f(x)=\begin{cases} kx^2e^{-x}, x>0 \\ 0, otherwise \end{cases}$. Find the mean and variance.

(OR)

11(b) (i) Derive the m.g.f of poisson distribution and hence deduce its mean and variance.

(ii) The time in hours required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$

(a)what is the probability that the repair time exceeds 2 hours?

(b) what is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours ?

12(a)(i) If the density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax & , 0 \le x \le 1 \\ a & , 1 \le x \le 2 \\ 3a-ax, & 2 \le x \le 3 \\ 0 & ,otherwise \end{cases}$$

(i) Find a (ii) Find the cdf of X.

(ii)Find the MGF of the random variable with the probability law $P(X=x)=q^{x-1}p, x=1,2,3,....$Find the mean and variance.

(OR)

(b)(i) Find the probability that in tossing a fair coin 5 times,there will appear (i) 3 heads (ii) 3 tails and 2 heads (iii)atleast 1 head (iv) not more than 1 tail.

(ii) A random variable X is uniformly distributed over (0,10). Find

$P(X<3), P(X>7), P(2<X<5)$ and $P(X \le 7)$.

13(a)(i)A random variable X has density function given by

$$f(x)=\begin{cases} \frac{1}{k} & , for\ 0<x<k \\ 0, & otherwise \end{cases}$$

Find (i) MGF (ii) mean (iii) variance.

(ii)Derive memoryless property of exponential distribution

(OR)

(b)(i) Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide-sense stationary ,Where A and ω are constants and θ is uniformly distributed on the interval $(0,2\pi)$.

(ii) Define a poisson process.show that the sum of two poisson processes is a poisson process.

14(a)(i)The process X(t) whose probability distribution under certain conditions is giveh by

$$p\{x(t) = n\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,\ldots\ldots \\ \dfrac{at}{1+at}, n = 0 \end{cases}$$ show that it is not stationary

(ii) Let the two random processes X(t) and Y(t) be defined as $X(t) = A\cos\omega t + B\sin\omega t, Y(t) = B\cos\omega t - A\sin\omega t$,where A and B are random variables, ω is a constants.If E(A)=E(B)=0 and $E(A^2)=E(B^2)$,prove that X(t) and Y(t) are jointly WSS.

(OR)

(b)(i) Consider the process X(t)= $A\cos\omega t + B\sin\omega t$,where A and B are R.V's with E(A)=0=E(B) and E(AB)=0.prove that X(t) is mean ergodic

(ii) The transition probability matrix of a markov chain X(t),n=1,2,3,....... Having three states 1,2,3 is $\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = [0.7 \quad 0.2 \quad 0.1]$

find $p(x_2=3)$ and $p(x_3=2,x_2=3,x_1=3,x_0=2)$.

15(a)(i) The tpm of a Markov process $\{X_n\}$,n=1,2,3..... having 3 states 0,1 and 2 is

$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ and the initial distribution $P^{(0)} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ Find (i)

$P(X_3 = 2, X_2 = 1)$ (ii) $P(X_2 = 2, X_1 = 1, X_0 = 2)$ (iii) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$
(iii)$P(X_2 = 2)$

(ii)A random process X(t) is defined as $X(t) = A\cos(\omega t + \theta)$ θ and ω are constants and A is random variable.Determine whether X(t) is a wss or not.

(OR)

(b)State and prove poisson postulates of poisson process