ECE

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JAYA GROUP OF INSTITUTIONS

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4^{TH}-SEMSETER-B.E

MODEL EXAMINATION-II (05.02.2015 TO 11.02.2015)

SUB CODE: MA6451 DATE :05.01.2015

SUBJECT TITLE: PROBABILITY AND RANDOM PROCESSES BRANCH : ECE

DURATION : 3 HRS MAX.MARK:100

PART-A (10*2=20)

Answer all the questions.

1.The joint P.D.F of R.V X and Y is given by $f(x,y) = Kxye^{-(x^2+y^2)}$,x>0,y>0.Find the value of K.

2.Let X and Y have j.d.f f(x,y)=2,0<x<y<1.Find the M.d.f.

3.If the joint p.d.f of (X,Y) is $f(x,y) = \frac{1}{4}$,o<x,y<2,find $P(X+Y \leq 1)$.

4.State the equation of the two regression lines.What is the angle between them?

5.Let X be a random variable with p.d.f $f(x) = \frac{1}{2}, \ -1 \leq x \leq 1$ and let $Y = X^2$.Find E(Y).

6.Define cross-spectral density.

7.Check whether the following functions are valid auto correlation function $\frac{1}{1+9\tau^2}$.

8.Find the mean square value of the random process whose Auto correlation is $\frac{A^2}{2}\cos\omega\tau$.

9.Is $\frac{\omega^2+4}{4\omega^4+3\omega^2+5}$ a valid power density spectrum?

10.State any two properties of an autocorrelation function.

PART-B (5*16=80)

11(a)(i) If the joint pdf of (X,Y) is given by P(x,y)=K(2x+3y),x=0,1,2:y=1,2,3.Find all the marginal probability distribution.Also find the probability distribution of (X+Y) and P(X+Y>3).

(ii) Show that the function $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y), 0<x<1,\ 0<y<1 \\ 0 \qquad otherwise \end{cases}$ is a joint density function of X and Y.

(OR)

(b)(i) The joint pdf of a two dimensional random variable (X,Y) is given by

$f(x,y) = xy^2 + \frac{x^2}{8} \quad 0 \leq x \leq 2, 0 \leq y \leq 1$ Compute

(a) $P(x > 1/y < \frac{1}{2})$ (b) $P(y < \frac{1}{2}/x > 1)$ (c) P(X<Y) (d) $P(X+Y \leq 1)$.

(ii)Given $f(x,y)=\begin{cases} Cx(x-y) & ,0<x<2,-x<y<x \\ 0 & ,otherwise \end{cases}$

(i) Find C

(ii) Find f(x)

(iii) Find f(y/x)

12(a)(i)Let X and Y be two random variables with the joint pdf $f(x,y)=\begin{cases} 8xy & ,0<x<y<1 \\ 0 & otherwise \end{cases}$

(i)Find the marginal probability density functions of X and Y.

(ii) Find $P(X<\frac{1}{4}/\frac{1}{2}<y<\frac{3}{4})$

(iii) Verify whether X and Y are independent

(iv) Find E(X) and E(Y).

(ii) The two lines of regression are 8x-10y+66=0,40x-18y-214=0

Find (i) The mean values of X and Y.

(ii) Correlation coefficient between X and Y.

(OR)

(b)(i) Two R.V's X and Y are related as Y=4X+9.Find the correlation coefficient between X and Y.

(ii) The joint pdf of X and Y is given by $f(x,y)=e^{-(x+y)}$,x>0,y>0, find the p.d.f of $U=\frac{X+Y}{2}$.

13(a)(i) Let X and Y be discrete R.V's with p.d.f $f(x,y)=\frac{x+y}{21}$ x=1,2,3 ; y=1,2

Find (i) Mean and variane of X and Y

(ii) Cov(X,Y)

(iii) $\rho(X,Y)$.

(ii)Obtain the rank correlation for the following data

X	68	64	75	50	64	80	75	40	55	64
Y	62	58	68	45	81	60	68	48	50	70

(OR)

(b)(i) The autocorrelation function for a stationary process is given by $R_{XX}(\tau)=9+2e^{-|\tau|}$ .Find the mean value of the random variable $Y=\int_0^2 X(t)\,dt$ and variance of X(t).

(ii) Show that the random process $X(t)=A\sin(\omega t+\phi)$,where A and ω are constants, ϕ is a random variable uniformly distributed in $(0,2\pi)$.Find the auto correlation function of the process.

14(a)(i) If $X(t) = 3\cos(\omega t + \theta)$ and $Y(t) = 2\cos(\omega t + \theta - \frac{\pi}{2})$ are two random processes where θ is a random variable uniformly distributed in $(0,2\pi)$ prove that $\sqrt{R_{XX(0)} R_{YY(0)}} \geq |R_{XY}(\tau)|$.

(ii) Find the mean square value of the processes whose power spectral density is as given below. $\frac{1}{\omega^4 + 10\omega^2 + 9}$.

(OR)

(b)(i) State and prove Wiener-Khinchine Theorem.

(ii) Find the power spectral density function whose Auto correlation function is given by $R_{XX}(\tau) = \frac{A^2}{2}\cos(\omega_0 \tau)$.

(OR)

15(a)(i)For the process $X(t) = A\cos(\omega_0 t + \theta)$ where A and ω_0 are real constants and θ is uniformly distributed in $(0, \frac{\pi}{2})$,find the average power P_{XX} in X(t).

(ii) If the power spectral density of a WSS process is given by $S_{XX}(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|) & , |\omega| \leq a \\ o & , |\omega| > a \end{cases}$ find the auto correlation function of the process.

(OR)

(b) (i)If the cross-correlation of two processes $\{X(t)\}$ and $\{Y(t)\}$ is $R_{XY}(t, t+\tau) = \frac{AB}{2}[\sin(\omega_0 \tau) + \cos(\omega_0(2t + \tau))]$ where A,B are ω_0 and constants.Find the cross power spectrum.

(ii)Consider the ergodic random process whose Auto correlation is $R_{XX}(\tau) = \begin{cases} 1 - |\tau| , |\tau| \leq 1 \\ 0 \quad , |\tau| > 1 \end{cases}$ .Find its spectral density.