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CSE/IT

JAYA GROUP OF INSTITUTION- Thiruninravur

4th sem – B.E

INTERNAL ASSESSMENT – II

Sub Name: PROBABILITY AND QUEUEING THEORY — **Date : 05:03:15**

Sub Code: MA6251 — **Branch : IT/CSE**

Duration: 3 hrs — **Max. Marks: 100**

PART A (10x2=20)

Answer All the Questions

1. Give a real life example each for positive correlation and negative correlation.
2. When will the two regression lines be (a) at right angles (b) coincident?
3. Let the joint pdf of the random variable (X,Y) be given by $f(x,y) = Kxye^{-(x^2+y^2)}; x>0 \text{ and } y>0$. Find the value of K.
4. Given the RV X with density function $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & elsewhere\end{cases}$. Find the pdf $Y=8X^3$
5. Given the two regression lines $3X+12Y=19$, $3Y+9X=46$, find the coefficient of correlation between X and Y
6. Define steady state and transient state in Queuing theory.
7. Define Markovian Queueing models.
8. Give a real life situation in which (a) customers are considered for service with last in first out queue discipline (b) a system with infinite number of servers.
9. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 mins between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 mins. Find the average number of persons waiting in the system.
10. What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queuing model represent?

PART – B (5*16=80)

Answer All The Questions

11. a) (i) The joint distribution of X and Y is given by $f(x,y)=\dfrac{x+y}{21}, x=1,2,3; y=1,2$. Find the marginal distribution. [8 *Marks*]

(ii)The joint probability density function of a two-dimensional random variable (X,Y) is $f(x,y)=\frac{1}{8}(6-x-y), 0<x<2, 2<y<4$. Find

I. $P(X<1 \cap Y<3)$

II. $P(X+Y<3)$

III. $P(X<1/Y<3)$. [8 *Marks*]

(OR)

b) (i) For two random variable X and Y with the same mean, the two regression Equations are $y = ax + b$ *and* $x = cy + d$. Find the common mean, ratio of the Standard deviations and also show that $\frac{b}{d} = \frac{1-a}{1-c}$. [8 *Marks*]

(ii) Obtain the equations of the lines of regression from the following data: [8 *Marks*]

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

12. a) (i) If X and Y are independent RVs with pdf's $e^{-x}, x \geq 0$, and $e^{-y}, y \geq 0$, respectively, find the density functions of $U = \frac{X}{X+Y}$ *and* $V = X + Y$. Are U and V independent? [8 *Marks*]

(ii) Find the correlation coefficient for the following data: [8 *Marks*]

X:	10	14	18	22	26	30
Y:	18	12	24	6	30	36

(OR)

b)(i) Given $f(x,y) = cx(x-y), 0 < x < 2, -x < y < x$ and '0' elsewhere. Evaluate ' c' and find $f_X(x)$ *and* $f_Y(y)$ respectively. [8 *Marks*]

(ii) If the joint pdf of (X,Y) be given by $f(x,y) = 2$, in $0 \leq x < y \leq 1$, find $E(X)$. [8 *Marks*]

13. a) (i) Two dimensional random variables (X,Y) have the joint probability density function $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}$ (I)Find $P\left[X < \frac{1}{2} \cap Y < \frac{1}{4}\right]$

(II) Find the marginal and conditional distributions.

(III)Are X and Y independent? [8 *Marks*]

(ii) Two random variables X and Y have the joint probability density function $f(x,y) = \begin{cases} c(4-x-y), 0 \leq x \leq 2, 0 \leq y \leq 2 \\ 0, \quad \textit{elsewhere} \end{cases}$. Find $\text{cov}(X,Y)$. [8 *Marks*]

(OR)

b) (i) Obtain the steady state probabilities of birth-death process. Also draw the transition graph. [8 *Marks*]

(ii) Calculate any four measures of effectiveness of M/M/1 queuing model. [8 *Marks*]

14. a)If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it takes exactly 1.5 min to reach the correct seat after purchasing the ticket,

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start of the picture?

(iii) How early must he arrive in order to be 99% sure of being seated for the start of the picture? [16 *Marks*]

(OR)

b) At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20 kms down the river. Tankers arrive according to Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average, 10 hrs to unload a tanker, the unloading time following an exponential distribution. Find

(i) how many tankers are at the port on the average?

(ii) how long does a tanker spend at the port on the average?

(iii) what is the average arrival rate at the overflow [16 *Marks*]

15. a) (i) Show that for the $(M/M/1):(FCFS/\infty/\infty)$, the distribution of waiting time in the system is $w(t)=(\mu-\lambda)e^{-(\mu-\lambda)t}, t>0$. [8 *Marks*]

(ii) A duplicating machine maintained for office use is operated by an office assistant who earns Rs.5 per hour. The time to complete each job baries according to an exponential distribution with mean 6 min. Assume a Poisson input with an average arrival rate of 5 jobs per hour. If an 8 hour day is used as a base, determine

(a) the % idle time of the machine

(b) the average time a job is in the system and

(c) the average earning per day of the assistant [8 *Marks*]

(OR)

b) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

i. Find the effective arrival rate at the clinic.

ii. What is the probability that an arriving patient will not wait?

iii. What is the expected waiting time until a patient is discharged from the clinic? [16 *Marks*]