PROBABILITY AND QUEUEING THEORY

UNIT - I RANDOM VARIABLES

(IMPORTANT UNIVERSITY PROBLEMS)

Discrete and continuous random variable – Moments, Moment generating functions and their properties. Binomial, Poisson, Geometric, Uniform, Exponential, Gamma and Normal distributions – Function of Random variable.

1. If the pdf of a random variable X is $f(x) = \frac{x}{2} in 0 \le x \le 2, find p(x > 1.5 / x > 1)$
2. Find the value of c given the pdf of a random variable X as $f(x) = \begin{cases} \frac{c}{x^3} & if\ 1 < x < \infty \end{cases}$.
3. If the c d f of RV is given by f(x)= $\begin{cases} \frac{x^2}{16} & for\ 0 \le x \le 4\ and\ find\ p(x > 1 / x < 3) \end{cases}$
4. The probability that a candidate can pass in an exam is 0.6.

(a) What is the probability that he pass in the third trial?

(b) What is the probability that he pass before the third trial?

8. Define exponential density function and mean and variance of the same.
9. Design moment generating function and write the formula to find mean and variance
10. Define central moments of random variable.
11. Write the moment generating function of Geometric distribution.
12. Write a note on functions of a random variable.
13. Find the moment generating function of binomial distribution.
14. Find the mean and variance of Geometric distribution.

. 15. A random variable Has the following distribution

x	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

(i) Evaluate k
(ii) Evaluate P(-2<x<3)
(iii) Find the cumulative distribution function of X .

16. A continuous random variable has the pdf $\lambda\ for\ which\ p(X \le \lambda) > \frac{1}{2}$
17. If $p(X = x) = \{kx,\ x = 1, 2, 3, 4, 5.\ 0\ for\ otherwise\ represents\ a\ pdf$

(i) Find the value of k
(ii) Find p(x being a prime number)

(iii) Find $p\left\{\frac{1}{2} < x < \frac{5}{2} / x > 1\right\}$

(iv) Find the distributive function.

18. The probability mass function of a random variable x is given by

$p(i) = \frac{c\lambda^i}{i!}(i = 0,1,2,....) whare \lambda > 0 Find$

(i) p (x=0) , (ii) p(x>2).

19. The time (in hours) required representing to repairs, a machine is exponential distribution with parameter $\lambda = \frac{1}{2}$.

(i) What is the probability that the repair time exceeds 2 hours?

(ii) What is the conditional probability that a repair takes at least 10 hour given that its duration exceeds 9 hours?

20. An irregular 6 faced dice is such that the probability that it gives 3 odd numbers in 7 throws is twice the probability that it gives 4 odd numbers in 7 throws. How many sets of exactly 7 trials can be expected to give no odd number out of 5000 sets?

21. Define Geometric distribution. Obtain its MGF and hence compute the first four moments.

22. For a normal distribution with mean 2 and variance 9, find the value of x_1 of the variable such the probability of the variable lying in the interval (2, x_1) is 0.4115.

23. A random variable X has a uniform distribution over the interval (-3,3).compute

(i) P(X=2)

(ii) $p(|X - 2| < 2)$

(iii) Find k such that p(x>k) =1/3.

24. Define Gamma distribution. Prove that sum of independent Gamma variates is Gamma variate .

25. In a book of 520 pages, 390 typographical errors occur .Assuming poison law for the number of errors per pages; find the probability that a random sample of 5 pages will contain no error.

26. A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

a. Find the value of k
b. P(1.5<x<4.5/x>2)and
c. The smallest value of λ *for which* $p(X \le \lambda) > \frac{1}{2}$

27. Prove that Poisson distribution is the limiting case of Binomial distribution.

28. Define Gamma distribution and find mean and variance of the same.

29. In an engineering examination a student is considered to have failed, secured second class and distinction, according as he scores less than 45% and 60%, between 60%and 75%and above 75% respectively. In particular year 10%of the students failed in the

examination and 5% of the students get distinction .Find the percentage of students who got first class and second class. (Assume normal distribution of marks).

30 .Define Binomial distribution and find its mean and variance.

31. If the MGF of a uniform distribution for a random variable X is $\frac{1}{t}(e^{5t}-e^{4t})$ $Find\ E(x)$.

32. The probability mass function of a random variable X is defined as $p(X=0)=3c^2$, $p(X=1)=4C-10C^2, P(X=2)=5C-1,\ where\ c>0$, and p(X=r) =0, if r$\neq$0,1,2….Find (i) The value of c

(ii) P (0<x<2 / x>0)

(iii) The distribution function of x

(iv) The largest value of x for which F(x) <1/2.

33. If the probability that an applicant for a drives license will pass the road test on any given trials 0.8 .what is the probability that he will finally pass the test(i)On the fourth trial and (ii)In less than 4 trials?

34. The marks obtained by a number of students in a certain subject are assumed to be normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that two of them will have marks over 70?

35. Find the probability distribution of the total number of heads obtained in four tosses of a balanced coin . Hence obtain the MGF of X, mean of X and variance of X.

36. In a small library there are 1000 books among 500 are scientific. Among the scientific books are 100 which belong to Engineering. Three books are chosen at random , the chosen books being replaced each time. What is the probability of getting (i) all 3 scientific books (ii) 3 scientific books among which only one is an engineering books (iii) at least one of the three is an engineering books .

37. If X has the distribution function $F(x)=\begin{cases} 0 & x<1 \\ \frac{1}{3} & for\, 1\le x\le 4 \\ \frac{1}{2} & for\, 4\le x\le 6 \\ \frac{5}{6} & for\, 6\le x\le 10 \\ 1 & for\, x\ge 10 \end{cases}$ Find

(i) The probability distribution of X.
(ii) p (2<x<6)
(iii) Mean of X
(iv) Variance of X

38. State and explain the proprieties of Normal N$\left(\mu,\sigma^2\right)$

39. Define Poisson distribution and state any two instances where Poisson distribution may be successfully employed.

40. Show that for the uniform distribution f(x)=1/2a -a<x<a the moment generating function about origin is $\frac{\sinh at}{at}$

41. If the probability mass function of a RV X is given by $P(X=r)=kr^3 \quad, r=1,2,3,4$. Find (i) the value of k , (ii) $P\left(\frac{1}{2}<X<\frac{5}{2}/X>1\right)$ (iii) the mean and variance of X and (iv) the distribution function of X.

42. The density function of a random variable X is given by f(x)= kx(2-x) 0<x<2. Find k, mean, variance , and r th moment

43. If the moments of a random variable X are defined by $E(X^r)=0.6\,; r=1,2,3.....$ Show that P(X=0)=0.4 , P(X=1) =0.6 , $P(X\geq 2)=0$

44. State and prove memory less property in geometric distribution.

45. State and prove memory less property in exponential distribution.

46. Six dice are thrown 729 time . How many times do you expect at least three dice to shows 5 or 6 .

47. If X follows $B\left(3,\frac{1}{3}\right)$ *and* Y *follows* $B\left(5,\frac{1}{3}\right)$ *find* $P\left(X+Y\geq 1\right)$

48. In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probabilities that (i) All are good bulbs (ii) at most 3 are defective bulbs (iii) Exactly 3 are defective bulbs.

49. If X is a Poisson variate such that P(X=2)=9P(X=4)+90P(X=6) , Find the variance.

50. If X and Y are independent Poisson variates , show that the conditional distribution of X given X+Y is binomial.

51. The sum of two Poisson variates is a Poisson variate.

52. If X_1 *and* X_2 *his independent Poisson* var*iates showthat* X_1-X_2 not a Poisson variate.

53. VLSI chips, essential to running of a computer system failing accordance with a Poisson distribution with the rate of one chip in about 5 weeks. If there are two spare chips in hand, and if a new supply will arrive in 8 weeks. What is the probability that during the next 8 weeks the system will be down for a week or more, owing to a lack of chips.

54. A manufacturer of television sets knows that of an average 5% of his product is defective. He sales television in consignment of 100 and guarantees that not more than 4 sets will be defective. What if the probability that a television set will fail to meet the guaranteed quality?

55. Suppose that a trainee soldier a target in an independent fashion. If the probability that a target is shot on any one shot is 0.8. (i) What is the probability that the target would be hit on 6^{th} attempt? (ii) What is the probability that he takes him less that 5 shots? (iii) What is the probability that it takes him an even number of shots?

56. If the probability that an applicant for a drivers license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (a) on the fourth trial and (ii) in fewer than 4 trials?

57. X uniformly distributed with mean 1 and variance 4/3, find P(X<0).

58. The mileage which car owners get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40000km. Find the probabilities that one of these tires will last (i)at lest 20000km and (ii) at most 30000 km.
59. The time in hours required to repair a machine is exponential distributed with perimeter $\lambda = \frac{1}{2}$. (i) What is the probability that the repair time will exceeds 2h.
(ii) What is the conditional probability that a repair takes at least 10h given that its duration exceeds 9h?
60. If a continuous random variable X follows uniform distribution in the interval (0,2) and a continuous random variable Y follows exponential distribution with parameter α *find* α such that P(X<1)=P(Y<1).
61. In an examination a student passes if he secures 30% or more marks. He is placed in the first, second or third division accordingly, as he secures 60%or more marks, between 45% and 60%marks and marks between 30% and 45% respectively. He gets a distinction in case he secures 80% or more marks. It is noticed from the results that 10% of the students failed in examination; where as 5% of them obtained distinction. Calculate the percentage of students placed in the second division.
62.In the experiment of rolling a balanced die twice, let X be the sum of the two number obtained. Determine the probability mass function of X.

63.If a R.V of X has the PDF $f(x) = \begin{cases} \frac{1}{4} & |x| < 2 \\ 0 & otherwise \end{cases}$

$(i)\, P(X < 1), \quad (ii) P(|X| > 1), \quad (iii) P(2X + 3 > 5)$

64.Two dice are thrown 120 times; Find the average number of times in which the number on the first die exceeds the number on the second die.
65.A bus arrives every 20 minutes, at a specified stop, beginning at a specified stop, beginning at 6.40 am. And continuing until 8.40 am. A passenger arrives randomly between 7.00 am and 7.30 am. What is the probability that the passenger has to wait for more than 5 minutes for a bus?
66.A man and a woman agree to meet at a certain place between 10 A.M. and 11 A.M. They agree that the one arriving first will wait 15 min for the other to arrive. Assuming that the arrival times are independent and uniformly distributed, find the probability that they meet.
67. The cdf of a continuous random variable is given by

$F(x) = \begin{cases} 0 & , X < 0 \\ 1 - e^{-\frac{x}{5}}, & 0 \le x < \infty \end{cases}$ .Find the PDF and mean of X.

68. The probability function of an infinite discrete distribution is given by
$P[X = x] = \frac{1}{2^j}\ (j = 1, 2, 3, 4.....).$ *Find* (i) mean of x (ii) p(X is even) (iii) p(X is divisible by 3).

69. A continuous random variable X has the pdf $f(x)=\begin{cases}\dfrac{k}{1+x^2} & ,-\infty < x < \infty \\ 0 & ,otherwise\end{cases}$. Find (i) the value of k (ii) Distribution function of x. (iii) $p(x \geq 0)$.

70. Let X and Y be independent normal variates with mean 45 and 44 and standard deviation 2 and 1.5 respectively. What is the probability that random chosen values of X and Y differ by 1.5 or more.

UNIT-2 TWO DIMENTIONAL RANDOM VARIABLES

(IMPORTANT UNIVERSITY QUESTIONS)

Joint distributions-Marginal and Conditional distributions Covariance-Correlation and regression – Transformations of random variables-Central limit theorem.

1. If the function f(x, y) =c (1-x) (1-y), 0<x<1, 0<y<1, to be a density function, find the value of c.
2. A random variable X has the pdf f(x)=$\begin{cases} e^{-x} & ,x>0 \\ 0 & ,x<0\end{cases}$ $.Find\ the\ dencity\ function\ of\ \dfrac{1}{x}$
3. If the joint pdf of (X , Y) is given by f(x,y)=2-x-y , in 0<X<1,0<Y<1.Find the marginal density of X and Y.
4. The joint probability mass function of (X , Y) is given by p(x,y)=k(2x+3y) , x =0,1,2; y=1,2,3; Find all the marginal and conditional probability distributions. Also find the probability distribution of (X+Y).
5. The joint probability density function of the Random variable (X, Y) is given by $f(x,y)=kxye^{-(x^2+y^2)}$ $,x>0, y>0$. Find the value of K and prove that X and Y are independent.
6. Find the marginal and conditional densities if $f(x,y)=k(x^3y+xy^3)$ $,0 \leq x \leq 2$ $,0 \leq y \leq 2$
7. The joint distribution of (X,Y) where X and Y are discrete is given in the following table

X	Y		
	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

Verify whether X and Y independent.

10. The joint pdf of a two dimensional random variable (X,Y) is given by $f(x,y)=\begin{cases} x^2+\dfrac{xy}{3}, & 0<x<1, 0<y<2 \\ 0 & elsewhere. \end{cases}$

(i) $P(X>\frac{1}{2})$ (ii) P(Y<X). (iii) $P\left(Y<\frac{1}{2} / X<\frac{1}{2}\right)$

11.If $f(x,y)=e^{-(x+y)}$, $x\ge 0$ $y\ge 0$ is the joint pdf of x and y , find p(x<1)

12. If f(x, y) =8xy, 0<x<1, 0<y<x is the joint pdf of x and y, find f(x/y).

13. Given the joint probability density function of (X, Y) $f(X,Y)=e^{-(x+y)}$, $x>0, y>0$

Find the marginal densities of X and Y. Are X and Y independent?

14. Given the joint probability density $f(x,y)=\begin{cases} \dfrac{2}{3}(x+2y) & for\ 0<x<1, 0<y<1, \\ 0 & otherwise \end{cases}$ Find the marginal densities, conditional density of X given by Y=y and $P\left(X\le\frac{1}{2} / Y=\frac{1}{2}\right)$.

15. The joint probability density function of a two dimensional random variable (X,Y) is given by

$f(x,y)=xy^2+\frac{x^2}{8}$, $0\le x\le 2$, $0\le y\le 1$. *Compute* P(X>1)

$P(Y<\frac{1}{2})$, $P\left(X>1/Y<\frac{1}{2}\right)$, $P\left(Y<\frac{1}{2}/X>1\right)$, $P(X<Y)$, $P(X+Y\le 1)$

16. Let X and Y have joint density function of f(x,y)=2, 0<x<y<1. Find the Marginal density function. Find the conditional density function of Y given X=x.

17. The joint density function of two random X and Y is given by $f(x,y)=\begin{cases} \dfrac{1}{3}(3x^2+xy), & 0<x<1, 0<y<2 \\ 0 & otherwise \end{cases}$ *Find* $P(X+Y\ge 1)$

18. The joint probability distribution of X and Y is given by

$f(x,y)=\left\{\dfrac{6-x-y}{8}\right.$ $0<x<2$, $2<y<4$. *Find* $f(y/x=2)$.

19. If the joint probability density function of random variable (X,Y) is given by

$f(x,y)=\begin{cases} x^2+\dfrac{xy}{3} & 0\le x\le 1, 0\le y\le 2 \\ 0 & otherwise \end{cases}$ .Find the conditional densities of X on Y and Y on X.

20. Calculate the correlation coefficient for the following heights (in inches) of fathers X and sons Y .

X	65	66	67	67	68	69	70	72

Y	67	68	65	68	72	72	69	71

21. Two random variables X and Y have the following joint probability density function. $f(x,y)=\begin{cases}2-x-y & 0\le x\le 1\ ,0\le y\le 1\\ 0 & otherwise\end{cases}$.Find Var(X), Var(Y) and also the covariance between X and Y.

22. Find the coefficient of correlation between industrial production and export using the following data:

Production	55	56	58	59	60	60	62
Export	35	38	37	39	44	43	44

23. If the joint probability density function of X and Y is

$f(x,y)=\begin{cases}e^{-(x+y)} & for\ x>0, y>0\\ 0 & ,otherwise\end{cases}$ Find the probability density function of $Z=\dfrac{X}{X+Y}$

24. Determine the value of C that makes the function F(x, y) =C(x+y) a joint probability density function over the range 0<x<3 and x<y<x+2. Also find the following (I) P(X<1.Y<2) (ii) P(Y>2) (iii) E(X).

26. Obtain the equation of the regression lines from the following data, using the method of least square. Hence find the coefficient of correlation between X and Y. Also estimate the value of Y when X=38 and the value of X when Y=18.

X	22	26	29	30	31	33	34	35
Y	20	20	21	29	27	24	27	31

27. If the joint distribution function of X and Y is given by $F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}) & for\ x>0, y>0\\ 0 & otherwise\end{cases}$

(i) Find the marginal densities of X and Y

(ii) Are X and Y independent

(iii)P (1<X<3, 1<Y<2)

28. The two lines of regression are 8x-10y+66=0, 40x-18y+214=0. The variance of X is 9.Find (i) the mean values of X and Y (ii) Correlation coefficient between X and Y.

29. If there is no linear correlation between two random variables X and Y, then what can you say about the regression lines.

30. Given

$f(x,y)=\begin{cases}cx(x-y) & ,0<x<2,-x<y<x\\ 0 & ,otherwise\end{cases}$ $.Evalute\, c\, and\ find\ f_x(x)\, and\ f_y(y)$.

31. Compute the correlation coefficient between X and Y using the following data

X	1	3	5	7	8	9
Y	8	12	15	17	18	20

32 For two random variables X and Y the same mean, the two regression equations are y=ax+b and x=cy+d.Find the common mean, ratio of the standard deviations and also show that $\dfrac{b}{d}=\dfrac{1-a}{1-c}$

33.The joint distribution of X and Y is given by f(x, y) = $\frac{x+y}{21}$, x=1, 2, 3 ; y=1, 2.Find the marginal distributions of X and Y

34. Find the covariance of X and Y if the random variable (X,Y)has the joint probability density function of f(x,y)=x+y, 0<x<1,0<y<1.

35. For 10 observations on price X and supply Y the following data is obtained $\sum X = 130, \sum Y = 220, \sum X^2 = 2288, \sum Y^2 = 5506 \, and \sum XY = 3467$. Obtain the line of regression of Y and X and estimate the supply when the price is 16 units.

36. If two dimensional random variables X and Y are Uniformly distributed in $0 \le X \le Y < 1$ find (i) correlation coefficient r_{xy} , (ii) Regression equation.

37. If the joint pdf of a two dimensional random variable X and Y is given by $f(x, y) = k(6 - x - y) \; 0 < x < 2, 2 < y < 4$. Find (i) the value of k (ii) P(x<1,y<3), (iii)P(x<1/y<3), (iv)P(y<3,/x<1), (v) P(x+y<3).

38. The marks secured by the recruits in the selection test X and in the proficiency test Y are given below calculate the rank correlation co-efficient

X	10	15	12	17	13	16	24	14	22
Y	30	42	45	46	33	34	40	35	39

39. Ten competitors in beauty contest are ranked by three judges in the following order

Competitor	1	2	3	4	5	6	7	8	9	10
Judge A	1	6	5	10	3	2	4	9	7	8
Judge B	3	5	8	4	7	10	2	1	6	9
Judge C	6	4	9	8	1	2	3	10	5	7

Using rank correlation co-efficient determine which pair of judges have common taste in beauty.

40. Marks obtained by 10 students in Mathematics (x) and Statistics (y) are given below

X	60	34	40	50	45	40	22	43	42	64
Y	75	32	33	40	45	33	12	30	34	51

Find the two regression lines. Also find y when x=55.

41. The Joint probability density function of a two dimensional random variables (X,Y) is given by $f(x, y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x \ge 0, y \ge 0 \\ 0 & , otherwise \end{cases}$. Find the density function of $U = \sqrt{X^2 + Y^2}$.

42. The independent random variables X and Y follow exponential distribution with parameter $\lambda = 1$.Find the probability density function of U=X-Y.

43. The joint probability density function of a two dimensional random variable (X, Y) is given by f(x,y)=x+y , $0 \le X, Y \le 1$. *Find the pdf of* $U = XY$.

44. If X and Y are independent random variables which are uniformly distributed over (0, 1), find the distributions of U=X+Y , V=X-Y.

45. Find the value of k if f(x,y)= kxy 0<x,y<1 is to be density function.

46. Prove that Cov(x,y)=0 , if X and Y are independent.

47. Two independent random variables X and Y are defined by $f(x)=\begin{cases}4ax, 0\le x\le 1\\ 0 \quad ,otherwise\end{cases}$, $f(y)=\begin{cases}4ay, 0\le y\le 1\\ 0 \quad ,otherwise\end{cases}$.Show that U=X+Y and V=X-Y are uncorrelated.

FORMULAE

- For Joint probability mass function

  Conditional probability distribution $P(X/Y)=\dfrac{P(X=x,Y=y)}{p(Y=y)}$

  Independent of random variable of X and Y $P_{ij}=P_{i*}\ X\ P_{*j}$

- For Joint probability density function

  Marginal density function of X is $f_X(x)=\int\limits_{-\infty}^{\infty} f(x,y)dy$

  Marginal density function of Y is $f_Y(y)=\int\limits_{-\infty}^{\infty} f(x,y)dx$

  Conditional probability density of X given Y is $f(X/Y)=\dfrac{f(X,Y)}{f_Y(y)}$

  Conditional probability density of Y given X is $f(Y/X)=\dfrac{f(X,Y)}{f_X(x)}$

  Independent random variable of X and Y $f(X,Y)=f_X(x)\,X\,f_Y(y)$

- Cov(X,Y)=E(XY)-E(X)E(Y)
- Correlation coefficient $r(X,Y)=\dfrac{E(XY)-E(X)E(Y)}{\sqrt{E(X^2)-(E(X))^2}\quad\sqrt{E(Y^2)-((E(Y))^2}}$
- Rank correlation $r=1-\dfrac{6\sum\limits_i d_i^{\,2}}{n(n^2-1)}$ $Where, d_i=X_i-Y_i$
- E(X) = $\int\limits_{-\infty}^{\infty} xf_X(x)dx$,E(Y) = $\int\limits_{-\infty}^{\infty} yf_y(y)dy$,E(XY) = $\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} xyf(x,y)dydx$

  $E(X^2)=\int\limits_{-\infty}^{\infty} x^2 f_X(x)dx$, $E(Y^2)=\int\limits_{-\infty}^{\infty} y^2 f_Y(y)dy$.

Regression Lines

- The line of regression Y on X is given by $Y-\overline{Y}=b_{yx}(X-\overline{X})$
- The line of regression X on Y is given by $X-\overline{X}=b_{xy}(Y-\overline{Y})$
- Correlation coefficient $r=\pm\sqrt{b_{xy}\ X\ b_{yx}}$.

UNIT-3 MARKOV PROCESS
(IMPORTANT UNIVERSITY PROBLEMS)

Definition and examples- first order, second order, strictly stationary, wide sense Stationary and Ergodic processes – Markov process – Binomial, Poisson and Normal Processes – Sine wave processes- Random telegraph process.

1. Define random process. Classify it with an example.
2. If $X(t) = Y\cos\omega t + Z\sin\omega t$, where Y and Z are two independent normal RVs with $E(Y) = E(Z) = 0, E(Y^2) = E(Z^2) = \sigma^2$ and W is constant, prove that {X (t)} is a SSS process of order 2.
3. State the postulates of a Poisson process and derive its probability law.
4. Prove that the sum of two independent Poisson process is a Poisson process.
5. Define strict sense stationary process and wide sense stationary process.
6. Define Markov process.
7. Given a random variable Y with characteristic function $\phi(\omega)$ *and a random process* $X(t) = \cos(\lambda t + Y)$ .Show that {X(t)} is stationary is the wide sense if $\phi(1) = 0$ *and* $\varphi(2) = 0$
8. Three boys X, Y, Z are throwing a ball to each other. X always throws the ball to Y and Y always throws the ball to Z, but Z is just as likely throw the ball to Y as to X. Show that the process is Markovian. Find the transition probability matrix and classify the states.
9. A machine goes out of order whenever a component fails . The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in next 10 weeks.
10. If $x(t) = A\sin(\omega t + \theta)$, where A and W are constants and θ and is RV uniformly distributed over $(-\pi, \pi)$ .Find the auto correlation of {y(t)},where y(t)= $x^2(t)$. $f(\theta) = \begin{cases} \dfrac{1}{2\pi}, & -\pi < \theta < \pi \\ 0 & , otherwise \end{cases}$
11. For a Random process x (t) =y$\sin\omega t$, y is uniformly distributed random variable in the interval (-1, 1). Check whether the process is wide sense stationary or not.
12. Determine whether the given matrix is irreducible or not $\begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$
13. Show that the random process $X(t) = A\cos(\omega_0 t + \theta)$ is not stationary. If A and ω_0 are constants and θ is uniformly distributed in $(0, \pi)$.

14. Define the stationary stochastic process. If $\{X(t), t\varepsilon T\}$ is a process with probability distribution $P(X(T)=n)=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}} & ,n=1,2,3...\\ \dfrac{at}{1+at} & ,n=0\end{cases}$ verify whether {X(t)} is a stationary process.
15. Write detailed notes on normal process.
16. Distinguish between stationary and weekly stationary stochastic processes. Give an example of each type. Show that Poisson process is an evolutionary process.
17. Write a critical note on sine wave process and its application.
18. Show that the random process $X(t)=A\cos(\omega_0 t+\theta)$ is not stationary. If A and ω_0 are constants and θ is uniformly distributed in $(0, 2\pi)$.
19. Define Poisson process and derive the probability law for Poisson process {X (t)}.
20. A man either drives a car or catches the train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he just as likely to drive again as he is to travel by train . Now he supposes that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii)the probability that he drives to work in the long run.
21. What is a stochastic matrix? When is it said to be regular?
22. Two random process X(t) and Y(t) are defined by $X(t)=A\cos\omega_0 t+B\sin\omega_0 t \; and \; Y(t)=B\cos\omega_0 t-A\sin\omega_0 t$. Show that X(t) and Y(t) are jointly wide-sense stationary , if A and B are uncorrelated random variables with zero means and same variances and ω_0 is a constant
23. A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses , find the transition probability matrix P of the Markov Chain $\{X_n, n\geq 0\}$. *Find also* P^2 *and* $P\{X_2=6\}$.
24. Assume that the number of messages input to a communication channel in an interval of duration t seconds , is a Poisson process with mean $\lambda=0.3$.compute

    (a) The probability that exactly 3 messages will arrive during the 10 seconds interval

    (b) The probability that number of messages arrivals in an interval of duration 5 seconds is between 3 and 7.

25. The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3, 4.....$ having the three states 1,2,3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$, and the initial distribution is $P^{(0)} = [0.7, 0.2, 0.1]$.Find $P(X_2 = 3)$ *and* $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

26. The random Binary transmission process {X(t)} is a wide sense process with T is a constant. Find the mean and auto correlation function $R(\tau) = 1 - \frac{|\tau|}{T}$, where T is a constant. Find the mean and variance of time average of {X (t)} over (0, T). Is {X (t)} mean – ergodic?

27. If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2/min, find the probability that the interval between 2 consecutive arrivals is more than 1 min , between 1 and 2 mins , and 4 mins or less.

28. An engineer analyzing a series of digital signal generated by a testing system observes that only 1 out of 15 highly distorted signals follow a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that ate highly distorted.

29. Suppose that a mouse is moving inside the maze shown in the adjacent figure from one cell to another, in search of food. When at a cell the mouse will move to one of the adjoining cells randomly. For $n \geq 0$. X_n Be the cell number the moves will visit after having changed cells n times. Is $\{X_n ; n = 0, 1, 2....\}$ a Markov chain? If so, write its state space and transition probability matrix.

1	4	7
2	5	8
3	6	9

30. The following is the transition probability matrix of Markov chain with state space { 0, 1, 2, 3, 4 }. Specify the classes, and determine which classes are

transient and which are recurrent. Give reasons

$$P = \begin{bmatrix} 2/5 & 0 & 0 & 3/5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 0 & 0 & 3/4 & 0 \\ 0 & 0 & 1/3 & 0 & 2/3 \end{bmatrix}$$

31. If [X(t)] is a WSS process with auto correlation function $R_{XX}(\tau)$ and if y(t)=X(t+a)-X(t-a). show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$.

32. If X(t)=Ycost+Zsint for all t where Y and Z are independent binary random variables , each of which assumes the values -1 and 2 with probability 2/3 and 1/3 respectively, prove that {X(t)} is wide sense stationery

33. Suppose that the probability of a dry day following a rainy day is 1/3 and that the probability of a rainy day following a dry day is 1/2. Given that May 1 is a dry day. Find the probability that May 3 is a dry day and also may 5 is a dry day.

34. Define irreducible Markov chain? And state Chapman-kolmogrow theorem.

35. A raining process is considered as two state Markov chain. If it rains it is considered to be state 0 and If it does not rain, the chain is in state 1. The transition probability of the Markov chain is defined as $\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$. Find the probability that it will rain for 3 days from today assuming that it will rain after 3 days. Assuming the initial probabilities of the state 0 and state 1 as 0.4 and 0.6 respectively.

36. A housewife buys 3 kinds of cereals A, B, C. She never buys the same cereals in successive weeks. If she buys cereal A, the next week she buys cereals B. However if she buys B or C the next week she is 3 times as likely to buy A as the other cereal. How often she buys each of the 3 cereals?

37. The difference of two independent Poisson process is not a Poisson process.

38. Suppose that customer arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min (a) exactly 4 customers arrive (b) more than 4 customers arrive.

UNIT – IV QUEUEING MODELS
(IMPORTANT UNIVERSITY PROBLEMS)

1. Consider an M/M/1 queuing system. If $\lambda = 6$ and $\mu = 8$, find the probability of at least 10 customers in the system.
2. Consider an M/M/C queuing system. Find the probability that an arriving customer is forced to join the queue.
3. Write Little's formula for the queuing model M/M/1/∞/FIFO.
4. For (M/M/1):(∞/FIFO) model, write down the Little's formula.
5. For (M/M/C):(N/FIFO) model, write down the formula for
   a) Average number of customers in the queue.
   b) Average waiting time in the system.
6. In a given M/M/1/∞/FCFS queue, $\rho = 0.6$, what is the probability that the queue contains 5 or more customers?
7. What is the effective arrival rate for M/M/1/4/FCFS queuing model when $\lambda = 2$ and $\mu = 5$.
8. What is the probability that a customer has to wait more than 15 minutes to get his service completed in (M/M/1):(∞/FIFO) queue system if $\lambda = 6$ per hour and $\mu = 10$ per hour?
9. What is the probability that an arrival to an infinite capacity 3 server Poisson queuing system with $\frac{\lambda}{\mu} = 2$ and $P_o = \frac{1}{9}$ enters the service without waiting?
10. In the usual notation of an M/M/1 queuing system if $\lambda = 3$/hour and $\mu = 4$/hour, find $P(X \geq 5)$, where X is the number of customers in the system.
11. Find $P(X \geq c+n)$ for M/M/c queuing system.
12. There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters are being typed at the rate of 15 letters per hour. What is the fraction of time all the typists will be busy?
13. What is traffic intensity of an M/M/1 queuing model? And define M/G/1 queue.
14. What is the effect arrival rate for M/M/1/N queuing system? What is the condition for steady state in terms of the traffic intensity?
15. What do you mean by transient state and steady state queuing system?
16. An automatic machine at the airport dispenses a cup of coffee in one minute. Customers arrive according to Poisson process with mean rate of 30 per hour. What is the average number of people that will be waiting for service?
17. Consider an M/M/1 queuing system if $\lambda = 6$ and $\mu = 8$, find the probability at least 10 customers in the system.
18. Consider an M/M/c queuing system. Find the probability that an arriving customer is forced to join the queue.

19. What is the probability that an arrival to an infinite capacity 3 server Poisson queuing system with $\frac{\lambda}{\mu} = 2$ and $P_0 = \frac{1}{9}$ enters the service without waiting?
20. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a M/M/1 queuing system, if $\lambda = 6$ per hour and μ = 10 per hour?
21. What are the basic characteristics of queuing process?
22. In a railway marshalling yard, goods train arrive at a rate of 30 trains per day. Assuming the inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the man queue size and the probability that queue size exceeds 10.
23. People arrive at a theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is constant at 2 minutes. Calculate mean number in waiting line and mean waiting line.

PART – B

1. Customers arrive at a watch repair shop according to a Poisson process at rate of one per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. (1).Find the average number of customers L_s in the shop

   (1).Find the average time a customer spends in the shop W_S

   (2).Find the average number of customer in the queue L_q

   (3).What is the Probability that the server is idle

   (4). Mean waiting time of a customer in the queue W_q
2. Find the average number of customers L_s in the M/M/I/N queuing system when $\lambda = \mu$
3. A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation only 4 cars are accepted for servicing. The arrival process is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ car per day per bay. Find the average number of cars in the service station and the average time a car spends in the system.
4. Obtain the expressions for steady state probabilities of a M /M /C queuing system.
5. Arrivals at a telephone booth are considered to be Poisson with an average time of 12min- between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min. Find the average numberofpersonswaitinginthe system .What is the probability that a person arriving at the book will have to wait in the queue? Also estimate the fraction of the day when phone will in use.

6. There are three typists in an office. Each typist can type an average of 6 letters per hour .If letters arrive for being typed at the rate of 15 letters per hour, what fraction of time all the typists will be busy?. What is the average number of letters waiting to be typed? What is the average time a letter has to spend for waiting and for, being typed? (2006/M)
   (1).What is the Probability that no letters are there in the system?
   (2). what is the probability that all the typists are busy?
7. A petrol pump station has 2 pumps. The service times follow the exponential distribution with mean of 4 minutes and cars arrive for service is a Poisson Process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service. What is the probability that the pumps remain idle?
8. Obtain the steady state probabilities for M /M / 1 /N /FCFS queuing model.
9. In a given M / M / 1 queuing system, the average arrivals is 4 customers per minute $\rho = 0.7$ .What is (1) mean number of customers L_s in the system (2). Mean number of customers L_q in the queue (3).Probability that the server is idle (4) mean waiting time W_S in the system.
10. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to distribute exponentially with mean 4 minutes.
    (A).Find the average number of customers waiting in the system.
    (B).What is the probability that a person arriving at the booth will have to wait in the queue?
    (C).Estimate the fraction of the day when the phone will be in use.
    (D).What is the mean queue length?
11. An average of 10 cars per hour arrives at a single-server drive-in teller. Assume that the average service time for each customer is 4 minutes, and both inter arrival times and services are exponential (1).What is the probability that the teller is idle (2).What is the average number of cars waiting in the line for teller? (3).What is the average amount of time a drive-in customer spends in the bank parking (4).On the average, how many customer per hour will be served by the teller?
12. A one –man barber has a total of 10 seats.Inter arrival times are exponentially distributed, and an average of 20 prospective customers arrive each hour at the shop. Those customers who find the shop full do not enter. The barber takes an average of 12 minutes to cut each customer's hair. Haircut times are exponentially distributed (1) on the average, how much time will be spent in the shop by customer who enters?
13. Customers arrive at a one Window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window

including that for the serviced car can accommodate a maximum of 3 cars other can wait outside this space.

(1).What is the probability that an arriving customer can drive directly to the space in front of the window?

(2).What is the probability that an arriving customer will have to wait out side the indicated space?

(3).How long the arriving customer is expected to wait before starting service?

14. An electricity band has 3 bill counters providing service exponentially distributed at the rate of 12 customers per hour. If receives on the average 24 customer per hour, in a Poisson distribution. Determine.

(1).The probability that a customer will be sent immediately.

(2).Find the probability that a customer will have to wait.

(3).What is the average total time that a customer must spend at the bill counter?

15. At a public Telephone booth in a post office arrivals one considered to be Poisson with an average inter-arrival time of 12 minutes. The length of the phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the followings

(1).What is the probability that a fresh arrival will not have to wait for the phone?

(2).What is the probability that an arrival will have to wait more than 10 minutes before the phone is free?

(3).What is the average length of Queues formed from time to time?

16. A super market has two girls ringing up sales at the counters. If the service time for each customer is exponential with 4 minutes and if people arrive in a Poisson fashion at the counter at the rate of 10 per hour, then calculate the following

(1).The probability of having to wait for service.

(2).The expected percentage of idle time each girls.

(3).If a customer has to wait finds the expected length of his waiting time.

17. A T.V Repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs set in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 sets per 8 hours day,

(1). what is the repairman’s expected Idle time each day?

(2). How many jobs are ahead on the average for the set just brought in?

(3).What is the average waiting time of a new arrival in the system?

18. A telephone exchange has two long distance operators. The telephone company finds that during the peak load, Long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.

(1).What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?

(2).If the subscribers will wait and are served in turn, what is the expected waiting time?

19. A concentrator receives messages from a group of terminals and transmits them over a single transmission line .Suppose that message arrives according to Poisson Process at a rate of one message every 4 milliseconds and suppose that message transmission times are exponentially distributed with mean 3 ms. Find the mean number of messages in the system and the mean total delay in the system. What percentage increase in arrival rate results in a doubling of the above mean total delay?
20. Discuss the M / M /1 queuing system with finite capacity and obtain its steady-state probabilities and the mean number of customers in the system.
21. A petrol pump station has 2 pumps. The service times follow the exponential distribution with mean of 4 minutes and cars arrive for service is a Poisson process at the rate of 10 cars per hour. Find the probability that a customer has to wait for service .What is the Probability that the pumps remain idle?
22. Define Kendall's notation. What are the assumptions are made for simplest queuing model.
23. Arrival rate of telephone calls at telephone booth are according to Poisson distribution with an average time of 12 minutes between two consecutive calls arrival. The length of telephone call is assumed to be exponentially distributed with mean 4 minutes.

(1).Determine the Probability that person arriving at the booth will have to wait.

(2).Find the average queue length that is formed from time to time.

(3).The telephone company will install second booth when convinced that an arrival would expect to have to wait at least 5 minutes for the phone. Find the increase in flows of arrivals which will justify a second booth.

(4).What is the probability that an arrival will have to wait for more than 15 min before the phone is free?

28. Arrival rate of telephone calls at telephone booth are according to Poisson distributionwith an average time of 12 minutes between one arrival and the next. The length of telephone call is assumed to be exponentially distributed with mean 4 minutes.

(1).What is the average number of customer in the system?

(2).What fraction of the day the phone will be in use?

(3).What is the probability that an arriving customer have to wait?

29. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more then 14 patients. Examination time per patient is exponential with mean rate of 20 per

hour.

(1).What is the probability that an arriving patient will not wait?

(2).What is the effective arrival rate?

30. Explain an M/M/1, Finite capacity queuing model and obtain expressions for the steady state probability for the system size.

31. For the steady state M/M/1 queuing model, prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$

32. On every Sunday morning, a Dental hospital renders free dental service to the patients. As per the hospital rules, 3 dentists who are equally qualified and experienced will be on duty then. If takes on an average 10 mins for a patient to get treatment and actual time taken is known to vary approximately exponentially around this average .The patients arrive according to the Poisson distribution with an average of 12 per hour. The hospital management wants to investigate the following:

(1).The expected number of Patients waiting in the queue.

(2).The average time that a patient spends at the hospital.

33. Self-service system is followed in a super market at a metropolis. The customer arrival occur according to a Poisson distribution with mean 40 per hour. Service time per customer is exponentially distributed with mean 6 mins.

(1).Find the expected number of customers in the system.

(2).What is the percentage of time that the facility is idle?

34. A branch of a national bank has only one typist. Since the typing work varies in length, the typing rate is randomly distributed approximating Poisson distribution with mean rate of 8 letters per hour. The letter arrive at a rate of 5 per hour during the entire 8 hour work day. If the typewrite is valued at Rs. 1.50 per hour. Determine equipment utilization, the percent time an arriving letter has to wait, average system time and average idle time cost of the typewriter per day.

UNIT – V ADVANCED QUEUEING MODELS
(IMPORTANT UNIVERSITY PROBLEMS)

1. Define Pollaczek-Khintchine (P – K) formula and explain the notations

2. Automatic car wash facility operators with only one bay. Cars arrive according to a Poisson process at the rate 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes, determine

   (1). Mean number of customers in the system L_s

   (2).Mean number of customer in the queue L_q

(3).Mean waiting time of a customer in the system W_S

3.Derive Pollaczek-Khinchine formula for the average number of customers in the M/G/1Queuing system.

4.Write a detail notes for (i)open networks (ii) closed networks.

5,Define Bottleneck state in queueing series

6. Automatic car wash facility operators with only one bay. Cars arrive according to a Poisson process at the rate 4 cars per hour and may wait in the facility's parking lot if the bay is busy.find L_s , L_q , W_S

(i)the swrvice time is constant and equal to 10 minutes.
(ii)follos uniform distribution between 8 and 12 minutes.
(iii)follws normal distributions with mean 12 mins and SD of 3 mins.
Follws discrete distributions with values 4,8,15 mins with corresponding probabailities 0.2,0.6,0.2.

7. A one man barber shop taken exactly 25 mins to complete one haircut.If custemer arrive at the barber in a poission fashion at an average rate of one every 40 mins ,how long on the average a customer spends in the shop?also find the average time a customer must wait for service.

8.In abig factory there are a large number of operating machines and two squential repair shops,which do the service of the damaged machines exponentialy with respective rates of 1/hr and 2/hr .If the cumulative failure rate of all the machines in the factory is 0.5 /hr.find (i)the probability that both repair shops are idle ,(ii)the average number of machinesin the service section of the factory and(iii) the average repair time of the machine.

9.Write deaail notesseries queues or tandom queue or open jackon networks.