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CSE/IT

JAYA GROUP OF ENGINEERING COLLEGES, THIRUNINRAVUR.

MODEL EXAM-I

Department of mathematics

PROBABILITY AND QUEUEING THEORY

DEPT : CSE/IT TIME: 3 Hrs.

SEM/YEAR : IV / II MAX MARKS :100

PART-A (10X2=20)

1. A random variable X has the following probability function

x:	0	1	2	3	4	
P(x)	k	3k	5k	7k	9k	Find the value of k

2.For a binomial distribution mean is 6 and variance is 2.find the first two terms of the distribution.

3. Two dice are thrown 120 times; Find the average number of times in which the number on the first die exceeds the number on the second die.

4 .Find mean of the following mgf of $M_x(t) = \frac{e^{5t}-e^{4t}}{t}$

5.state and prove recurrence relation in poisson distribution

6. Prove that a first order stationary random process has a constant mean.

7.Define Poisson process.

8.Consider a Markov chain with 2 states and tpm P=$\begin{bmatrix}3/4 & 1/4\\1/2 & 1/2\end{bmatrix}$ .Find the steady state probabilities of the chain.

9.Define WSS process.

10.Define Markov process and give examples.

PART-B (5X16=80)

11 (a) A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

a. Find the value of k

b. P(1.5<x<4.5/x>2)and

c. The smallest value of λ *for which* $p(X \le \lambda) > \frac{1}{2}$

(b) Derive MGF, mean and variance of geometric distribution

(OR)

11(C)A manufacturer of television sets knows that of an average 5% of his product is defective. His sales television in consignment of 100 and guarantees that not more than 4 sets will be defective. What if the probability that a television set will fail to meet the guaranteed quality?

(d) Derive MGF, mean and variance of Gamma distribution.

12.(a)The probability function of an infinite discrete distribution is given by

$P[X=x]=\frac{1}{2^j}$ $(j=1,2,3,4.....).Find$ (i) mean of x (ii) p(X is even) (iii) p(X is divisible by 3).

(b) State and prove memory less property in exponential distribution.

(OR)

12.(C) . If a continuous random variable X follows uniform distribution in the interval (0,2) and a continuous random variable Y follows exponential distribution with parameter α $find\ \alpha$ such that P(X<1)=P(Y<1).

(d). Prove that Poisson distribution is the limiting case of Binomial distribution.

13.(a) Derive MGF,mean and variance of exponential distribution.

(b)The marks obtained by the students in maths,physics and chemistry is normally distributed with means 52,50,48 and SD 10,8,6 respectively . find the probability that a student selected at random has secured a total of (i)180 or above (ii)135 or less.

(OR)

13.(C) Show that the random process $X(t)=A\cos(\omega_0 t+\theta)$ is not stationary. If A and ω_0 are constants and θ is uniformly distributed in $(0,\pi)$.

(d) A man either drives a car or catches the train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he just as likely to drive again as he is to travel by train . Now he supposes that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii)the probability that he drives to work in the long run.

14.(a) Define the stationary stochastic process. If $\{X(t), t\varepsilon T\}$ is a process with probability distribution

$$P(X(T)=n)=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}} & ,n=1,2,3...\\[2ex] \dfrac{at}{1+at} & ,n=0\end{cases}$$

verify whether {X(t)} is a stationary process.

(b) The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3,4.....$ having the three states

1,2,3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$, and the initial distribution is $P^{(0)} = [0.7, 0.2, 0.1]$.Find

$P(X_2 = 3)$ *and* $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

(OR)

14.(C) Define Poisson process and derive the probability law for Poisson process $\{X(t)\}$.

(d) Three boys X, Y, Z are throwing a ball to each other. X always throws the ball to Y and Y always throws the ball to Z, but Z is just as likely throw the ball to Y as to X. Show that the process is Markovian. Find the transition probability matrix and classify the states.

15.(a) If $X(t) = Y\cos\omega t + Z\sin\omega t$, where Y and Z are two independent normal RVs with $E(Y) = E(Z) = 0, E(Y^2) = E(Z^2) = \sigma^2$ and W is constant, prove that {X (t)} is a WSS process

(b) A machine goes out of order whenever a component fails . The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in next 10 weeks

(OR)

15.(c) Two random process X(t) and Y(t) are defined by
$X(t) = A\cos\omega_0 t + B\sin\omega_0 t$ *and* $Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$.Show that X(t) and Y(t) are jointly wide-sense stationary , if A and B are uncorrelated random variables with zero means and same variances and ω_0 is a constant

(d) An engineer analyzing a series of digital signal generated by a testing system observes that only 1 out of 15 highly distorted signals follow a highly distorted signal, with no recognizable signal between, whereas 20 out of 23 recognizable signals follow recognizable signals, with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that ate highly distorted.