IT

JAYA GROUP OF INSTUTION – THIRUNINRAVUR

4TH SEMESTER – B.E./B.TECH

MODEL EXAMINATION – III

SUBJECT/SUBJECT CODE: PROBABILITY AND QUEUEING THEORY

DURATION: 3 HRS

CLASS: IT/CSE

DATE: 10/04/2015

PART – A (10*2=20)

1. If a random variable has the moment generating function given by $M_x(t)=\frac{2}{2-t}$ determine the variance of X.
2. Check whether the following is a probability density function or not: $f(x)=\begin{cases}\lambda e^{-\lambda x}, x\ge 0, \lambda>0\\ 0, \quad elsewhere\end{cases}$
3. The regression equations of X on Y and Y on X are respectively $5x-y=22$ and $64x-45y=24$. Find the means of X and Y.
4. If the joint pdf of (X,Y) is given by $f(x,y)=2; 0\le x<y\le 1$. find $E(X)$.
5. Define wss process.
6. Define Transition probability matrix.
7. Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains find the probability that the yard is empty.
8. What are the characteristics of a queuing system?
9. Define open network of a queuing system.
10. State Pollaczek – Khintchine formula.

PART – B (5*16=80)

11. (a) (i) A random variable X has the following probability function:

X	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i. Find the value of k.

ii. Evaluate $p(X<6)$, $p(X\ge 6)$.

iii. If $p(X \le c) > \frac{1}{2}$, find the minimum value of c

(a)(ii) Find the moment generating function of an exponential random variable and hence find its mean and variance.

(OR)

b) (i) A random variables X has the p.d.f $f(x)=\begin{cases}2x, & 0<x<1\\ 0, & otherwise\end{cases}$,

Find (i) $P\left(X<\frac{1}{2}\right)$,(ii) $P\left(\frac{1}{4}<X<\frac{1}{2}\right)$,(iii) $P\left(X>\frac{3}{4}/X>\frac{1}{2}\right)$,(iv) $P\left(X<\frac{3}{4}/X>\frac{1}{2}\right)$

(ii) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (1) on the 4th trial (2) in fewer than 4 trials?

12. a)The joint probability density function of a two-dimensional random variable (X,Y) is $f(x,y)=\frac{1}{8}(6-x-y), 0<x<2, 2<y<4$. Find (1) $P(X<1 \cap Y<3)$ (2) $P(X+Y<3)$ (3) $P(X<1/Y<3)$.

(OR)

(b)(i) Find the correlation coefficient for the following data:

X:	10	14	18	22	26	30
Y:	18	12	24	6	30	36

(ii) If X and Y are independent RVs with pdf's $e^{-x}, x \ge 0$, and $e^{-y}, y \ge 0$, respectively, find the density functions of $U=\frac{X}{X+Y}$ and $V=X+Y$. Are U and V independent?

13. (a)(i) The transition probability matrix of a Markov chain $\{X(t)\}, n=1,2,3,\ldots$, having three states 1,2 and 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $p^{(0)}=(0.7 \quad 0.2 \quad 0.1)$ Find $p[X_2=3], p[X_3=2, X_2=3, X_1=3, X_0=2]$,

(ii)Show that the random process $X(t)=A\cos(\omega_0 t+\theta)$ is wide - sense stationary, if A and ω_0 are constants and θ is uniformly distributed RV in $(0,2\pi)$.

(OR)

(b)(i) The process $\{X(t)\}$ whose probability distribution under certain condition is given by $P[X(t)=n]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots\\ \dfrac{at}{1+at}, & n=0\end{cases}$. Show that $\{X(t)\}$ is not stationary.

(b)(ii) Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes (1) Exactly 4 customers arrive (2) More than 4 customers arrive (3) less than 4 customers arrive.

14. (a)Customer arrive at a one man barber shop according to a Poisson process with a mean inter arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. The service time is exponentially distributed. If an hour is used as a unit of time, then

(i) What is the probability that a customer need not wait for a haircut?

(ii) What is the expected number of customer in the barber shop and in the queue?

(iii) How much time can a customer expect to spend in the barber shop?

(iv) Find the average time that a customer spends in the queue.

(v) Estimate the fraction of the day that the customer will be idle?

(vi) What is the probability that there will be 6 or more customers?

(vii) Estimate the percentage of customers who have to wait prior to getting into the barber's chair.

(OR)

(b)(i) A supermarket has 2 girls running up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, find the following

(1) What is the probability of having to wait for service?

(2) What is the expected percentage of idle time for each girl?

(3) What is the expected length of customer's waiting time?

(ii) Obtain the steady state probabilities of birth-death process. Also draw the transition graph.

15. (a) Derive the expected steady state system size for the single server queues with Poisson input and General Service.

(OR)

(b) (i)Consider two servers. An average of 8 customers per hour arrive from outside at server 1 and an average of 17 customers per hour arrive from outside at server 2. Inter arrival times are exponential. Server 1 can serve at an exponential rate of 20 customers per hour and server 2 can serve at an exponential rate of 30 customers per hour. After completing service at station 1, half the customers leave the system and half go to server 2. After completing service at station 2, 3/4 of the customer complete service and 1/4 return to server 1. Find the expected no. of customers at each server. Find the average time a customer spends in the system.

(b)(ii) Write short notes on the following :

(i) Queue networks

(ii) Series queues

(iii) Open networks

(iv) Closed networks